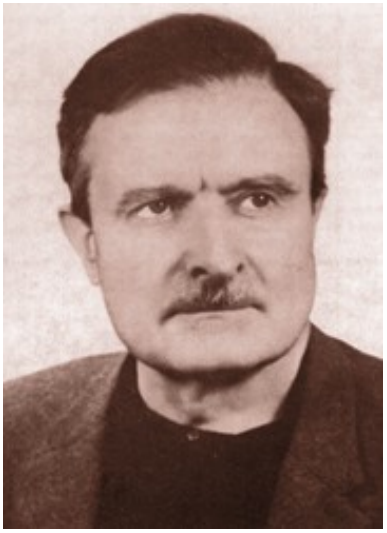
Łojasiewicz 100

International conference
to commemorate the centenary of
Professor Stanisław Łojasiewicz (1926-2002)

September 21st-25th, 2026

Booklet of abstracts

Jagiellonian University
Faculty of Mathematics and Computer Science
Kraków, Poland



Jagiellonian University, 2026

About the conference

Conference website: `lojasiewicz100.matinf.uj.edu.pl`

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Jagiellonian University, Faculty of Mathematics and Computer Science
Łojasiewicza 6, 30-348 Kraków, Poland

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Foreword

This conference is organized to commemorate the centenary of Professor **Stanisław Łojasiewicz** (1926–2002), whose pioneering work continues to shape modern mathematics and inspire generations of researchers. It focuses on recent developments in analytic geometry and singularity theory, reflecting the depth and breadth of Prof. Łojasiewicz's legacy. To provide PhD students, early-career researchers, and non-specialists with a substantive introduction to his research and its contemporary developments, the program features two dedicated mini-courses.

We wish you all a nice and fruitful stay in Kraków!

The organisers

Stanisław Łojasiewicz

Stanisław Łojasiewicz was born on October 9th, 1926 in Warsaw. Hesitating between music and mathematics, he eventually completed his mathematical studies in 1945-47 at the Jagiellonian University in Kraków. From that moment on he was connected with the Jagiellonian University, as well as with the Mathematical Institute of the Polish Academy of Sciences. The magnitude of his mathematical achievements sets him in the rank of the most outstanding mathematicians of the 20th century. His printed output counts 70 positions and concerns differential equations, theoretical mechanics, differential analysis, distribution theory and analytic geometry.

Under the influence of Tadeusz Ważewski he started his research in differential equations. He defended his PhD thesis entitled *Sur l'allure asymptotique des intégrales du système d'équations différentielles au voisinage de point singulier* in 1950. Then he turned his interests towards distribution theory created in the 50's by Laurent Schwartz. During his first stay abroad, in Paris in 1957, he solved Schwartz's problem of the division of a distributions by an analytic function. His solution, published in 1958 in CRAS had a great significance for the development of the theory of partial differential equations, differential analysis and analytic geometry. Łojasiewicz's solution was based on a fundamental growth property of real analytic function, known nowadays as *the Łojasiewicz inequality*. The method of the proof of this inequality led him to the creation of *semianalytic geometry* which soon evolved into *subanalytic geometry*, developed by A. Gabrielov, R. Hardt, H. Hironaka and others, dealing with proper images of semianalytic sets by analytic maps. Since many mathematical problems lead to studying such objects, semi- and subanalytic geometry, closely related to semialgebraic geometry, became a useful tool in many branches of analysis as approximation theory, control theory, and also in logic in connection with model theory. Interestingly, Łojasiewicz's seminal IHES preprint *Ensembles semi-analytiques* has never been officially published.

In 1970 Stanisław Łojasiewicz was invited to give a lecture about semianalytic geometry at the International Congress of Mathematicians in Nice. In 1971 he was elected to Polish Academy of Sciences as a correspondent member and became its full member in 1980. In 1983 he was elected to the Pontificia Academia Scientiarum.

Since the late 70's Stanisław Łojasiewicz was studying mainly subanalytic geometry, as well as complex analytic geometry to which he devoted his monograph *An introduction to complex analytic geometry*. This monograph as well as *An introduction to the theory of real functions* and the IHES preprint mentioned above are examples of mastery in mathematical precision and conciseness.

Stanisław Łojasiewicz passed away on November 14th, 2002.

Contents

Abstracts of minicourses

Łojasiewicz's gradient inequality: various aspects and applications

Sept. 21, 9:00
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Sept. 24, 9:00

Krzysztof Kurdyka
Savoie Mont Blanc University, France

This inequality has an important consequence: uniform bound for the length of gradient trajectories between levels of a potential. As a consequence trajectories have unique limits, which is particularly interesting when trajectory approaches a critical level.

I will discuss several instances in which the inequality, in its various forms, holds.

Lecture 1. Talweg lines

Background: semialgebraic and subanalytic geometry, o-minimal structures.

Łojasiewicz's inequality for functions definable in o-minimal structures.

Talweg lines (locus of local minima of the norm of gradient on the level of the potential), comparison principle: the length of a gradient trajectory bounded by the length of a talweg.

Applications: uniform bound for the length of trajectories of polynomials of bounded degree; convergence of proximal algorithm.

Lecture 2. Łojasiewicz's gradient inequality and KL-inequality

KL-inequality (Łojasiewicz inequality for functions definable in an o-minimal structure). Original Łojasiewicz's gradient inequality, quantitative version for polynomials (estimates for exponent depending on degree of polynomials).

Lecture 3. Thom's Gradient Conjecture and KL inequality for definable maps

Thom's Gradient Conjecture (existence of the limit of secants) and KL inequality for definable maps, open problems.

Topics in machine learning through o-minimal geometry

Sept. 21, 17:25
Sept. 22, 9:00
Sept. 25, 9:00

Jérôme Bolte

Toulouse School of Economics, France

We shall show how o-minimality provides a useful mathematical framework for some key phenomena in optimization and modern AI. In particular, we shall develop tools for automatic differentiation of nonsmooth definable programs and explain how the Baur-Strassen cheap gradient principle persists in the o-minimal setting. Nonsmooth Łojasiewicz inequalities will also be established and used to study the convergence of training and descent algorithms. Applications include projected methods, matrix factorization, and the training of neural networks. This geometric viewpoint also provides some insight into how neural networks can converge to interpolating solutions in generic situations.

Abstracts of invited lectures

Approximation of maps between real algebraic varieties

Sept. 21
9:55

Wojciech Kucharz
Jagiellonian University, Poland

A nonsingular real algebraic variety Y is said to have the approximation property if, for every nonsingular real algebraic variety X, every $\mathcal{C}^\infty$ map $f\colon X \to Y$ that is homotopic to a regular map can be approximated in the $\mathcal{C}^\infty$ topology by regular maps.

In this talk, I will present a characterization of nonsingular real algebraic varieties with the approximation property, obtained in joint work with Juliusz Banecki. I will also discuss two classes of examples: homogeneous spaces for linear algebraic groups and uniformly rational varieties.

Recent developments in real algebraic geometry

Sept. 21
14:20

Juliusz Banecki
Jagiellonian University, Poland

During my talk I will describe some new results concerning approximation of smooth manifolds by real algebraic sets. I will cover both positive results and counterexamples, including a recent solution to a problem of Nash. I will end the presentation with a very optimistic conjecture, which could solve the approximation problem in the general context once and for all.

Łojasiewicz inequalities and applications to gauge theory

Sept. 21
15:15

Paul M. N. Feehan
Rutgers University, USA

We shall survey geometric proofs of the Łojasiewicz inequalities, describe their generalization to infinite dimensions based on ideas of Leon Simon, and survey their applications to gauge theory, including Yang–Mills and Chern–Simons gradient flows.

On the Jacobian polygon and Łojasiewicz exponent of isolated complex hypersurface singularities

Sept. 21
16:30

Baldur Sigurðsson

Polytechnic University of Madrid, Spain

Given an isolated hypersurface singularity, we introduce an alternating version of Teissier's Jacobian Newton polygon, and prove formulas for both these invariants in terms of an embedded resolution. The alternating version has an advantage, in that for Newton nondegenerate functions, it can be calculated in terms of volumes of faces of the Newton diagram (similarly to Varchenko's formula for the monodromy zeta function), whereas a similar formula for the original non-alternating version includes mixed volumes. This work started from trying to prove a conjecture by Brzostowski, Krasiński and Oleksik, for which we provide a counterexample.

As a part of this work, we proved that Teissier's vanishing rates, associated with components of the polar curve, induce a filtration of a Morse–Smale complex associated with a Morse function on the Milnor fiber which appears naturally in this setting. We believe this result to be of independent interest.

Combinatorial type and splitting invariants of plane curves

Sept. 22
9:55

Taketo Shirane

Tokushima University, Japan

The study of the embedded topology of plane curves originated with the example given by Zariski in 1929. Since the complex projective plane is a real four-dimensional space and a plane curve is a real two-dimensional object, the embedded topology of plane curves may be regarded as an analogue of surface-knot theory. The fundamental group of the curve complement is a fundamental invariant of the embedded topology and has been studied extensively. Nevertheless, pairs of plane curves have been found that have the same combinatorial type, determined by the number of irreducible components and the types of singularities, and whose complements have isomorphic fundamental groups, but whose embedded topologies are different. Splitting invariants describe how a plane curve "splits" by the pull-back under a Galois cover of the complex projective plane whose branch locus contains no component of the plane curve. They enable us to distinguish the embedded topology of several plane curves that have isomorphic fundamental groups of their complements. In this talk, we introduce a generalization of splitting invariants, called the *G-combinatorial type* (or *G-combinatorics* for short), for plane curves by using the modified plumbing graph defined by Hironaka. We apply G-combinatorics to distinguish the embedded topology of quasi-triangular curves that cannot be distinguished by splitting invariants.

Singularities of Special Submanifolds in Symplectic Space

Sept. 22
14:20

Goo Ishikawa
Hokkaido University, Japan

The principal aim of this survey is to describe how methods from singularity theory interact with the geometry of Lagrangian and related symplectic subvarieties. Lagrangian, isotropic, and coisotropic subvarieties of symplectic manifolds play fundamental roles in symplectic geometry, geometric optics, classical and quantum mechanics, and related areas. In this survey, we discuss several central topics concerning these distinguished classes of subvarieties, including their stability, the characterisation of open Whitney umbrellas as an analogue of Mather's stability theory, symplectic reduction, generating families, and other closely related developments. We also review the basic differential-geometric techniques underlying the singularity theory of differentiable mappings. Furthermore, motivated by the study of symplectic singularities of planar frontals with well-defined tangent directions, we develop the singularity theory of frontals in symplectic manifolds. In particular, we introduce the notion of symplectic frontal stability for Lagrangian subvarieties. As an application, we obtain symplectic characterisations of the singularities known as open swallowtails, originally discovered and investigated by Arnol'd, Givental', and Janeczko, using our new concept of symplectic frontal stability. This is a joint work with Stanisław Janeczko.

Proof of the metric Arnold corank problem

Sept. 22
15:15

Zbigniew Jelonek
Polish Academy of Sciences, Poland

In this lecture, I approach the Arnold corank problem, posed by Arnold in 1975, which asks whether the corank of holomorphic functions is an ambient topological invariant. Here, we obtain a complete positive answer to the metric Arnold corank problem, which asks whether the corank of holomorphic functions is an ambient bi-Lipschitz invariant. Consequently, we show that for complex hypersurfaces, the multiplicity equal to two is an ambient bi-Lipschitz invariant. We also prove that the Arnold corank problem holds true for holomorphic functions of three variables. Other topological invariants are also presented. This is based on the common work with Alexandre Fernandes and José Edson Sampaio.

About the non-oscillation conjecture

Sept. 22
16:30

Vincent Grandjean
Federal University of Santa Catarina, Brazil

I will discuss the non-oscillation conjecture in dimension three. It is a joint work with Professor Tadeusz Mostowski.

Resolution of varieties and foliations by torus actions

Sept. 24
9:55

Jarosław Włodarczyk
Purdue University, USA

Classical resolution of singularities relies on blowing up smooth centers, which often complicates equations before improving them. A more natural approach assigns weights to coordinate directions to match the geometry of the singularity. While standard weighted blow-ups typically introduce difficult quotient singularities, we can bypass these issues using "cobordant blow-ups"—a construction utilizing torus actions where singularities are resolved simply by removing a vertex. This talk introduces the cobordant machinery from the ground up and extends it to foliated varieties. By properly aligning our blow-up centers with the foliation, we obtain a straightforward, algorithmic method for embedded resolution. We will conclude by demonstrating how this framework resolves totally integrable and Darboux foliations.

On a Variant of Jung's Approach to Resolution of Singularities in Positive Characteristic

Sept. 24
14:20

Hussein Mourtada
Paris Cité University, France

I will begin by recalling the Jungian approach to the resolution of singularities and explaining why, in positive characteristic, it is not clear whether this provides an effective framework for studying singularities and their resolution. I will then propose a variant of this approach in which the Teissier singularities, introduced recently in joint work with Bernd Schober, play a central role.

Perturbation of Polynomials and Linear Operators

Sept. 24
15:15

Adam Parusiński
Côte d'Azur University, France

The perturbation of polynomials and linear operators is a classical subject that began with Rellich's work in the 1930s. The parameter dependence of the polynomials (resp. operators) ranges from real analytic to C^∞ and differentiable of finite order, with drastically different regularity results often observed for the roots (resp. eigenvalues and eigenvectors). In this talk, I will present several recent results, such as an optimal Sobolev regularity estimate for the roots of monic complex polynomials of one variable with smooth dependence on a single real parameter; multiparameter versions of this result; and the continuity of the coefficients-to-roots map with respect to the C^d and Sobolev norms. Recently, Antonini, Cavalletti and Lerario reinterpreted some of these results in terms of the Wasserstein metric in order to study optimal transport between algebraic hypersurfaces in complex projective space. (Based on joint work with Armin Rainer.)

Fourier transform and complex analytic continuation of power-constructible functions

Sept. 24
16:30
online

Georges Comte
Savoie Mont Blanc University, France

The question of the belonging of parametric integrals of functions (in a given class or algebra of functions) to the same class, is in general a very difficult one. The additional question of the tameness of those integrals, with respect to the tameness of the functions we integrate is an other difficulty. I will describe an algebra of functions, containing the subanalytic functions, their logarithms and their (real or complex) powers, that is stable under parametric integration. Moreover there is a deep interplay between the asymptotic of the Fourier transform of the smooth and integrable functions of this algebra and the geometry of the complex domain to which they analytically extend. This is a joint work with D. Miller and T. Servi.

Łojasiewicz inequalities and quantitative uniqueness in mean curvature flow

Sept. 24
17:25
online

William Minicozzi
Massachusetts Institute of Technology, USA

In many geometric problems, singularities are analyzed using a blow analysis that constructs a limit solution from a sequence of dilations around a singularity. One of the central questions is whether this limit is unique or might it depend on the choice of rescaling?

This geometric problem is closely connected to celebrated work of Łojasiewicz. I will explain work with Toby Colding on this for mean curvature flow.

A theory to study metric degenerations: Moderately Discontinuous Algebraic Topology

Sept. 25
9:55

Maria Pe Pereira
Complutense University of Madrid, Spain

In the same way algebraic topology gives a language to talk about properties of topological spaces up to homeomorphism/homotopy we give a theory to talk about metric degenerations where the metric and dynamical information matters.

In the works [1] and [2] we developed a first version where we impose subanalyticity. In this talk I will explain the more general framework of continuous families of metric spaces we are working on.

In a second talk, Javier Fernández de Bobadilla will explain further results in the particular case of definable categories.

[1] with J. Fernández de Bobadilla, S. Heinze, E. Sampaio, *Moderately Discontinuous Homology*, Communications on Pure and Applied Mathematics, volume 75, Issue 10, p. 2123–2200. https://doi.org/10.1002/cpa.22013

[2] with J. Fernández de Bobadilla and S. Heinze, *Moderately Discontinuous Homotopy*. International Mathematics Research Notices, Volume 2022, Issue 23, December 2022, Pages 18346–18400. https://doi.org/10.1093/imrn/rnab225

Moderately discontinuous algebraic topology in the definable case

Sept. 25
11:10

Javier Fernández de Bobadilla

Basque Center for Applied Mathematics, Spain

In this talk, which will be a follow up of Maria Pe Pereira talk, I will explain further results on moderately discontinuous algebraic topology, that by now depend on definability in an o-minimal structure. I will sketch some proofs.

From the Jacobi-Perron algorithm to graded local uniformization

Sept. 25
12:05

Bernard Teissier

Paris Cité University

Graded local uniformization is the fact that given a field k and a totally ordered abelian group Φ of finite rational rank r, the semigroup $\Phi_{\geq 0}$ of non negative elements of Φ is the union of a nested sequence of free semigroups $\mathbb{N}^r$.

I will show how this follows from an extension of the Jacobi-Perron algorithm for approximating an arbitrary vector in $\mathbb{R}^n$ by regular rational convex cones, just as continued fractions do for $n = 2$.

Abstracts of contributed talks

Characteristic classes via Lipschitz stratifications.

Sept. 21
11:10

Jean-Paul Brasselet
Aix-Marseille University, France

Marie-Hélène Schwartz defined characteristic classes for singular complex analytic varieties within the framework of obstruction theory and Whitney stratifications. This construction employs complex techniques.

In joint work with Tadeusz Mostowski and Thuy Nguyen Thi Bich, we show how the use of Lipschitz stratifications (instead of Whitney stratifications) allows for simplification of the Marie-Hélène Schwartz's construction.

On Zariski pairs of surface singularities

Sept. 21
11:50

Öznur Turhan
University of Warsaw, Poland

A conjecture of S. Yau asserts that if two isolated surface singularities in $\mathbb{C}^3$ have the same monodromy zeta-function and the same abstract topology, then they must also share the same embedded topology. This conjecture was disproved by E. Artal Bartolo, who explicitly constructed a pair made of two superisolated surface singularities with identical zeta functions and abstract topologies but distinct embedded topologies. Today, any pair exhibiting this phenomenon is referred to as a Zariski pair of surface singularities. Subsequent research has further explored Zariski pair-like phenomena, resulting in several recent developments in the field. In this talk, we will present a concise introduction to this subject and its main results. This is a joint work with C. Eyral, M. Ishikawa and M. Oka.

Uniform Yomdin-Gromov Parametrizations in Valued Fieds with Analytic Structure.

Sept. 21
12:30

Krzysztof Nowak

Jagiellonian University, Poland

The paper concerns uniform Yomdin-Gromov parametrizations together with an estimate of their number, which generalizes a theorem by Cluckers-Forey-Loeser to arbitrary equicharacteristic zero valued fields with analytic structure. To this end, we establish a certain strong analytic stratification of definable sets, based on a definable non-Archimedean version of Bierstone-Milman's canonical desingularization algorithm from an earlier paper of ours. Other basic tools are: elimination of valued field quantifiers, term description of functions definable in analytic structures, and Lipschitz cell decomposition compatible with RV-parametrized sets in Hensel minimal structures.

Asymptotic lines and curvature lines defined by the Gauss map of surfaces in $\mathbb{R}^4$

Sept. 22
11:10

Wojciech Domitrz

Warsaw University of Technology, Poland

This is a joint work with L. I. Hernández-Martínez and F. Sánchez-Bringas. The Gauss map of a smooth, oriented surface M immersed in $\mathbb{R}^4$ takes values on the Grassmanian of oriented two-planes in $\mathbb{R}^4$. Since this manifold has a structure of a product of two spheres, this map has two components $g_i, ß = 1, 2$ that take values on the sphere. We define a new notion of asymptotic lines on M. Namely, the g_i-asymptotic lines which at each point p determine directions of higher contact (up to order two) of M with a complex curve that better approaches M near p. In the present research, we present the first study of the foliations defined by these directions near the simplest singularities denoted as g_i-parabolic points. Moreover, we define the notion of g-curvature lines; that is, curvature lines on M defined by the Gauss map. This set of geometric lines seems to be the most accurate generalization of the classic asymptotic and curvature lines defined on surfaces immersed in $\mathbb{R}^3$.

On Combinatorial Methods for Computing the Multiplicity of Overdetermined Map Germs

Sept. 22
11:50

Adam Różycki
University of Łódź, Poland

In this talk, we provide an effective formula for the multiplicity of a overdetermined, finite, non-degenerate analytic map germ in terms of the Newton diagrams of its components.

We also present a simple formula for the multiplicity in the equidetermined complex case. As further applications, we derive effective formulas for the Milnor number and some estimation for Tjurina number.

Moreover, we give a formula for the Łojasiewicz exponent in the case of gradient maps.

On nonregular Łojasiewicz numbers and polar invariants of plane curve singularities

Sept. 22
12:30

Andrzej Lenarcik
Kielce University of Technology, Poland

Let $f = 0$ be an isolated singularity defined by $f \in \mathbb{C}\{X, Y\}$ (f reduced). The Łojasiewicz exponent $\mathcal{L}_0(f)$ is the minimal possible exponent λ in the inequality $|\operatorname{grad} f(z)| \geq C|z|^\lambda$ for $C > 0$ near $0 \in \mathbb{C}^2$.

From Teissier's formula [T] it follows that

$$\mathcal{L}_0(f) = \sup_\gamma \frac{\operatorname{ord}(f \circ \gamma)}{\operatorname{ord}\gamma} - 1 \tag{1}$$

where $\gamma(T) \in \mathbb{C}\{T\}^2$, $\gamma(0) = 0 \in \mathbb{C}^2$, runs over a finite set of analytic arcs that parametrize a finite set of branches of a polar curve $a(\partial f/\partial X) + b(\partial f/\partial Y) = 0$ defined with respect to a generic direction $(a : b) \in \mathbb{P}^1(\mathbb{C})$. Hence (1) may be written as

$$\mathcal{L}_0(f) = \max\{q_1, \ldots, q_s\} - 1, \quad (s \geq 1). \tag{2}$$

Elements of the set $\mathcal{Q}(f) = \{q_1, \ldots, q_s\}$ are called polar invariants. Therefore the Łojasiewicz exponent of the gradient equals the maximal polar invariant decreased by 1. From (2) $\mathcal{L}_0(f)$ is a rational number. A. Płoski [P2] proved that $\mathcal{L}_0(f)$ is always in the set

$$\left\{1, 2, 3, 3\frac{1}{2}, 4, 4\frac{1}{3}, 4\frac{1}{2}, 4\frac{2}{3}, \ldots\right\} = \{1\} \cup \left\{N + \frac{b}{a} : \text{for integers } 0 \leq b < a < N\right\}. \tag{3}$$

A. Płoski with E. García Barroso [GB-P] showed that not every number from (3) is achieved by $\mathcal{L}_0(f)$. When T. Krasiński joined the team in [GB-K-P] they proved (applying [L1]) that $\mathcal{L}_0(f)$ from (3) satisfies additionally $a + b \leq N$ for f nondegenerate in the sense of Kouchnirenko [Kou]. They called the numbers in (3) with $a + b > N$ and $\operatorname{GCD}(a, b) = 1$ nonregular and asked whether or not a nonregular $\mathcal{L}_0(f)$ exists?

Analogously q is defined to be a nonregular polar invariant if the number $q-1$ is nonregular. They pointed the necessary limitations on the equisingularity class of singularities with a nonregular polar invariant (such invariant may be at most one). Unfortunately this nonregular polar invariant must not be the maximal one as they stated with $q_{\mathrm{nreg}}-1=169\frac{142}{143}$ that $\mathcal{L}_0(f)\geq 170$. This phenomenon makes it harder to study the set of possible values of nonregular Łojasiewicz numbers of singularities, which has not been yet characterized. Some partial results are presented in [GB-K-P]. For example a nonregular Łojasiewicz number $\lambda=N+\frac{b}{a}$ must have a composite $a>8$ and every such a is realized by a singularity. The minimal possible value is $\lambda=15\frac{8}{9}$.

During the proposed presentation a phenomenon $q_{\mathrm{nreg}}-1<\mathcal{L}_0(f)$ will be studied. As a result some sufficient conditions for $\lambda=(2\nu_1\nu_2-e)+(\nu_1\nu_2-h)/(\nu_1\nu_2)$, $(\nu_1,\nu_2>1, e>h>0$ integers) to be a nonregular Łojasiewicz number will be presented. The technique of Eggers–Płoski trees ([L2, L3]) obtained by using the logarithmic distance of Płoski ([P1, Ch-P]) will be applied. Such trees are a version of the classical Eggers approach ([E, GB, W1, W2]).

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Decorated Trees and Numerical Semigroups

Sept. 23
9:00

Pierrette Cassou-Noguès

University of Bordeaux, France

The link between the two subjects is classical if we think about semigroups associated to singularities of algebraic plane curves and their trees.

In this talk we want to enlarge the classes of objects that can be involved in the relation between the two topics.

Singularities in affine reflection and symmetrization of convex plane curves

Sept. 23
9:40

Stanisław Janeczko

Warsaw University of Technology, Poland

The new affine invariants of curves and surfaces are studied and in particular several point symmetrizations of a convex curve Γ are introduced and one, the affinely invariant "central symmetric transform" (CST) with respect to a given basepoint inside Γ, is investigated in detail.

Examples for Γ include triangles, rounded triangles, ellipses, curves defined by support functions and piecewise smooth curves.

Of particular interest is the region of basepoints for which the CST is convex (this region can be empty but its complement in the interior of Γ is never empty).

The (local) boundary of this region can have cusps and in principle it can be determined from a geometrical construction for the tangent direction to the CST.

Inner Lipschitz Geometry of Complex Surface Germs with Non-Isolated Singularities

Sept. 23
10:20

Yenni Cherik

Polish Academy of Sciences, Poland

A germ of a complex analytic set is a set $(X, 0)$ defined as the common zero locus of a finite collection of germs of holomorphic functions (or convergent power series) at the origin of $\mathbb{C}^n$.

In this talk, we will address the problem of classifying such objects. We will focus on the inner Lipschitz classification, which lies between analytic and topological classification.

The Lipschitz classification of complex curve germs was completed through the work of Pham and Teissier in 1969, followed by Neumann and Pichon in 2013. The Lipschitz classification of complex surface germs with isolated singularities with respect to the inner metric (the arc length distance) was completed by Birbrair, Neumann, and Pichon in 2014.

We will present a first step toward the inner Lipschitz classification of complex surface germs with non-isolated singularities. More precisely, we will prove that every complex surface germ with non-isolated singularities is inner bilipschitz equivalent to the quotient of a complex surface germ with an isolated singularity by an explicit equivalence relation, called (d)-curling.

On topological invariance of the Łojasiewicz exponent

Sept. 23
11:20

Tadeusz Krasiński

University of Łódź, Poland

We show how from known results it follows that the Łojasiewicz exponent of an isolated singularity is not a topological invariant.

Effective quasianalytic Remez and Łojasiewicz inequalities on tame sets

Sept. 23
12:00

Armin Rainer

Vienna University of Technology, Austria

The classical Remez inequality bounds the supremum norm of a polynomial on the unit interval in terms of its norm on a subset E of positive measure, with a constant depending only on the degree and the measure of E. It has been generalized to multidimensional convex bodies and sets with cusps.

We establish a Remez inequality for functions in quasianalytic Denjoy–Carleman classes $\mathcal{C}_M$ on a large family of compact fat sets with tame geometry, including all subanalytic ones. The polynomial degree is replaced by the Bang degree, an integer computed from the weight sequence M and the size of the function, and all constants are explicit in terms of the Bang degree and the geometry of the underlying set.

As a consequence, we derive effective Łojasiewicz inequalities with fully explicit exponents and constants, including a bound on the vanishing order and gradient inequalities. These results extend classical inequalities that are known to hold in polynomially bounded o-minimal structures but whose constants are typically unspecified.

Anisotropic deformation of Riemannian metrics leading to time geodesics in Finsler geometry

Sept. 23
12:40

Piotr Kopacz

Gdynia Maritime University, Poland

Deforming an arbitrary Riemannian metric by a vector field under some restrictions yields Finsler metric of (α, β)-type, for instance, Randers, Kropina, or Lorentz. Such a bijective transformation refers to the rigid translation of the corresponding indicatrix, and this is called the Zermelo navigation (viewed as the geometric method). Also, the length of any piecewise $\mathcal{C}^\infty$-curve in a Finsler manifold stands for the time for which the object travels along it.

Hence, the Finslerian geodesics model the time-minimizing paths under the action of the perturbing vector fields. In this talk, we will discuss an extension of the method, incorporating the anisotropy-based deformation. This gives rise to a general framework in Finsler geometry for unifying previous models for the navigation problems posed initially by Zermelo as well as Matsumoto. In particular, the approach being considered allows us to obtain and analyze the geodesics of the new Finsler metrics that belong to the class of general (α, β)-metrics, establishing the necessary and sufficient conditions for strong convexity.

The talk is based on joint work with Nicoleta Aldea.

Fast cycles of codimension 1 and thick-thin decomposition of complex surface germs. [Via coverings and discriminants]

Sept. 24
11:10

Dmitry Kerner

Ben Gurion University of the Negev, Israel

Take a surface germ X and its $\mathrm{Link}_r[X]$. When r goes to zero, $\mathrm{Link}_r[X]$ shrinks linearly "at most of its points". They form the thick zone of $\mathrm{Link}_r[X]$.

In "most cases" $\mathrm{Link}_r[X]$ has also thin zones, which shrink faster than linearly. These thin zones are fibred over fast cycles, subanalytic subsets of $\mathrm{Link}_r[X]$, of codimension 1.

Present X as a ramified covering of $(\mathbb{C}^2, o)$. The topological types of the fast cycles (and their vanishing rates) can be extracted from the discriminantal curve.

In particular this allows to distinguish (easily) various surface germs by their inner-Lipschitz type.

On Newton multipolyhedra and local analytic geometry

Sept. 24
11:50

Grzegorz Oleksik
Poznań University of Technology, Poland

In the 1975 problem list [Arn04], Vladimir I. Arnold posed two fundamental problems regarding the Newton polyhedron of a singularity:

1975-1 Every interesting discrete invariant of a generic singularity with a Newton polyhedron I is an interesting function of the polyhedron. Study: the signature, the number of moduli, the singularity index, the integral monodromy, the variation, the Bernstein polynomial, and μ_i (for generic section).

1975-21 Express the main numerical invariants of a typical singularity with a given Newton polyhedron (e.g., the signature, the genus of the 1-dimensional Milnor fiber) in terms of the polyhedron.

By a singularity, we mean a germ of a holomorphic function $f : (\mathbb{C}^n, 0) \longrightarrow (\mathbb{C}, 0)$ defined in a neighborhood of its critical point at the origin. The study of discrete invariants of complex analytic singularities often relies on extracting combinatorial and convex-geometric data from the associated Newton polyhedron in the real space $\mathbb{R}^n$. To date, numerous invariants have been successfully determined from the Newton polyhedron under suitable non-degeneracy conditions (i.e., for singularities with generic coefficients), most notably the Milnor number [Kush76]. Recently, the Łojasiewicz exponent of a singularity has also been expressed in terms of its Newton polyhedron (see [Sig26]).

In this talk, we extend Arnold's research program to the framework of local complex algebra. Let $I = (f_1, \ldots, f_m)$ be an ideal in the ring of holomorphic germs $\mathcal{O}_n$. We define the tuple of the Newton polyhedra of the generators f_i as the Newton multipolyhedron of the ideal I. Under an appropriate non-degeneracy condition, termed weak non-degeneracy, several discrete invariants of I can be read off directly from its Newton multipolyhedron. Recent advancements include the determination of the Lê numbers [EOR21] and the Krull dimension of the quotient ring $\mathcal{O}_n/I$ (see [Ole26]).

Motivated by these results, we propose the following generalizations of Arnold's original problems to the context of ideals:

2025-1 Express the main numerical invariants of a typical ideal with a given Newton multipolyhedron (e.g., the Krull dimension, the $\mathbb{C}$-dimension of quotient ring, Segre numbers, Lê numbers, Łojasiewicz exponent) in terms of the multipolyhedron.

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Area-Normalized Pentagram Map Dynamics: Spectral Flattening and Elliptic Asymptotics

Sept. 24
12:30

Michał Zwierzyński

Warsaw University of Technology, Poland

We study the dynamics of the pentagram map under an affine normalization that keeps the area of successive iterates fixed. Our approach is based on the spectral properties of the associated Glick operator, which determine the dominant geometric behaviour of normalized polygons. We prove generic flattening for strictly convex pentagons: the iterates converge to a line predicted by the dominant spectral directions. For convex hexagons, we establish a spectral dichotomy between flattening and bounded oscillatory behaviour asymptotic to families of ellipses. We also analyse projectively periodic polygons and construct explicit examples exhibiting higher-order return dynamics. Numerical experiments suggest that the same spectral mechanism governs more general polygons, although additional limiting lines and more complicated oscillations may occur. These results connect projective integrability of the pentagram map with affine asymptotic geometry.

Spectral Curves and Ordinary Differential Operators

Sept. 25
8:20

María Ángeles Zurro
Autonomous University of Madrid, Spain

This talk surveys the algebro-geometric structures associated with certain commutative subalgebras of ordinary differential operators (ODOs), a line of research to which I have contributed through the study of spectral curves for third-order operators and of spectral Picard–Vessiot fields for algebro-geometric Schrödinger operators [6, 5]. We begin with the foundational Burchnall–Chaundy theorem [1] and trace its geometric development toward non-planar spectral curves [6], together with recent computational frameworks for their study [4].

A differential Galois theory for these problems has only recently been developed, largely through our own work [5]. When the operators admit sufficiently regular real coefficients, however, this theory and its applications remain largely unexplored, save for the pioneering contributions of Crespo, Hajto, and van der Put [2, 3]. Building on this program, I will present a new theoretical result concerning the variational equation of the $(1+1)$-dimensional Korteweg–de Vries equation, extending the differential Galois approach to this setting.

This is joint work with J. J. Morales-Ruiz and J. P. Ramis.

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Poster

Invited Speakers

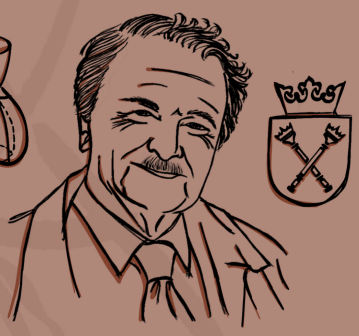
Juliusz Banecki (Uniwersytet Jagielloński)
Georges Comte (Université Savoie Mt. Blanc)
Paul M. N. Feehan (Rutgers University)
Javier Fernández de Bobadilla (BCAM)
Vincent Grandjean (Univ. Federal da Santa Catarina)
Goo Ishikawa (Hokkaido University)
Zbigniew Jelonek (IMPAN)
Wojciech Kucharz (Uniwersytet Jagielloński)
William Minicozzi (MIT)
Hussein Mourtada (Université Paris Cité)
Adam Parusiński (Université de Côte d'Azur)
Maria Pe Pereira (Universidad Complutense)
Taketo Shirane (University of Tokushima)
Baldur Sigurðsson (Univ. Politécnica de Madrid)
Bernard Teissier (Université Paris Cité)
Jarosław Włodarczyk (Purdue University)

Mini Courses

Jérôme Bolte
(Toulouse School of Economics)
Krzysztof Kurdyka
(Univ. Savoie Mt. Blanc)

ul. Prof. S. Łojasiewicza 6

Łojasiewicz100



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